Solution to supplementary problems.

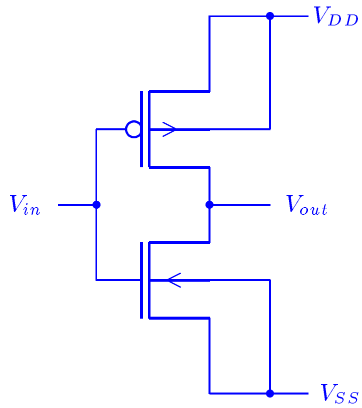
Problem 7

The threshold voltage, V_t, is defined as the voltage necessary in order to create strong inversion at the surface, i.e. to create a conductive channel. According to Formula (2.2), V_t is used to deplete the space charge in the depletion region, Q_B. As V_{SB} is increased, Q_B increases; as V_{SB} is decreased, Q_B decreases. Thus, Q_B must be increased, if a higher value of V_t is desired.

Warning: Formula (2.7) is only valid, when $V_{SB} > 0V$. For $V_{SB} < 0V$, the correct formula is

$$V_t = V_{t,0} + \gamma \left[\sqrt{2\phi_b + V_{SB}} - \sqrt{2\phi_b}\right]$$

The desired circuit diagram is shown to the right.



Problem 8

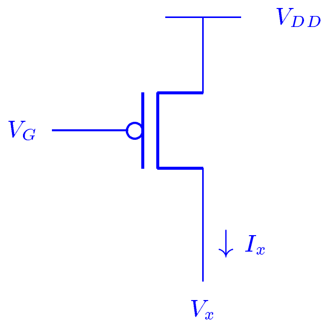
The p-channel transistor (shown to the right) must be a '*good current source*'. This will require that the current I_x is independent of the output voltage V_x.
Therefore, the transistor must operate in saturation. The transistor is saturated, when

$$\begin{aligned} 0\,V &> V_{GS} - V_{tp} > V_{DS} \quad \Leftrightarrow \\ V_{DD} &> V_G - V_{tp} > V_x \end{aligned}$$

E.g., V_G and V_x could be connected. This requires

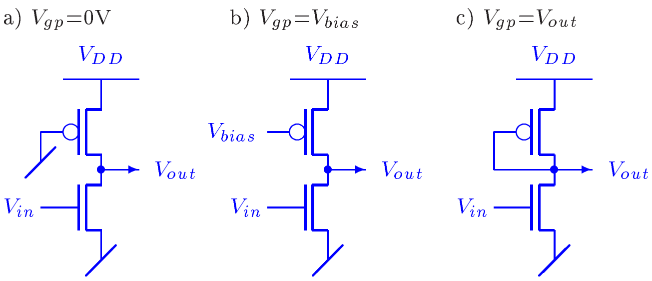
$$\begin{aligned} V_{DD} + V_{tp} &> V_x \Leftrightarrow \\ 1.32\,V &> V_x \end{aligned}$$

However, many other possibilities are possible!



Problem 9

Question 1:



Working ranges of the p-channel transistor

cut-off	$0 < V_{gs} - V_{tp}$
linear	$V_{gs} - V_{tp} \le V_{ds} \le 0V$
saturated	$V_{ds} \le V_{gs} - V_{tp}$

The working ranges of the p-channel transistor in configurations a), b) and c) are

Configuration a) $V_{gs} = -V_{DD}$; $\quad V_{ds} = V_{out} - V_{DD}$:
The p-channel transistor operates in the linear region, when $V_{out} \geq -V_{tp}$,
and the p-channel transistor is saturated, when $V_{out} \leq -V_{tp}$.

Configuration b) $V_{gs} = V_{bias} - V_{DD}$; $\quad V_{ds} = V_{out} - V_{DD}$:
The p-channel transistor operates in the linear region, when $V_{out} \geq V_{bias} - V_{tp}$,
and the p-channel transistor is saturated, when $V_{out} \leq V_{bias} - V_{tp}$.

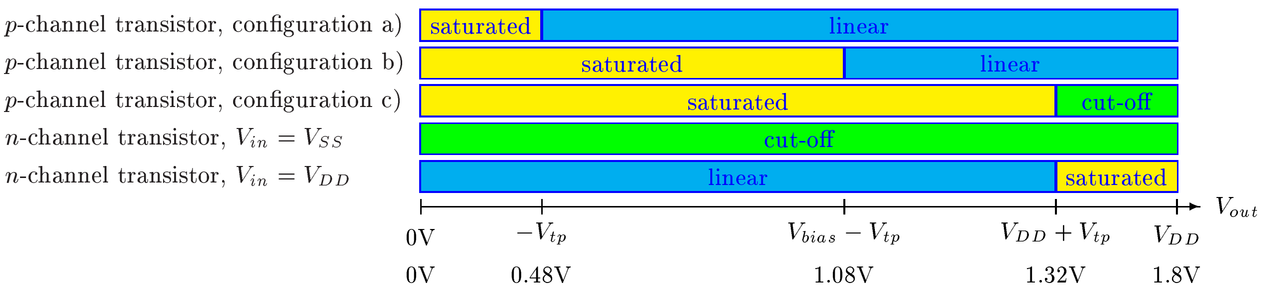
Configuration c) $V_{gs} = V_{ds} = V_{out} - V_{DD}$:
The p-channel transistor is cut-off, when $V_{out} > V_{DD} + V_{tp}$,
and the p-channel transistor is saturated, when $V_{out} \leq V_{DD} + V_{tp}$.

Working ranges of the n-channel transistor in states V_{in} low and V_{in} high are

V$_{in}$ low:
The n-channel transistor is cut-off.

V$_{in}$ high:
The n-channel transistor operates in the linear region, when $0V \leq V_{out} \leq V_{DD} - V_t$,
and the n-channel transistor is saturated, when $V_{DD} - V_t \leq V_{out} \leq V_{DD}$.

This is illustrated in the Figure below. Values of parameters are obtained from the design rules sheet for the 0.18μm process.



Question 2:

Calculation of V$_{outH}$) $V_{in} = 0V$:
In all 3 configurations, the n-channel transistor is cut-off. Thus, V_{outH} is determined by the p-channel transistor alone:
Configuration a) $V_{outH} = V_{DD} = 1.8V$
Configuration b) $V_{outH} = V_{DD} = 1.8V$
Configuration c) $V_{outH} = V_{DD} + V_{tp} = 1.32V$

Calculation of V$_{outL}$) $V_{in} = 1.8V$:
The following analysis applies to configurations a) and b). Assuming the n-channel transistor operates in the linear region, while the p-channel transistor is saturated:

$$\begin{aligned}
\frac{1}{2}\beta_p \left(V_{gsp} - V_{tp}\right)^2 &= \beta_n \left(V_{gsn} - V_{tn} - \frac{1}{2}V_{dsn}\right) V_{dsn} \Leftrightarrow \\
\frac{1}{2}\beta_p \left(V_{bias} - V_{DD} - V_{tp}\right)^2 &= \beta_n \left(V_{DD} - V_{tn} - \frac{1}{2}V_{out}\right) V_{out} \Leftrightarrow \\
\frac{1}{2}V_{out}^2 - (V_{DD} - V_{tn})V_{out} &+ \frac{1}{2}\frac{\beta_p}{\beta_n}\left(V_{bias} - (V_{DD} + V_{tp})\right)^2 = 0
\end{aligned}$$

Inserting the approximation $V_{tn} = -V_{tp} = V_t$:

$$V_{out} = (V_{DD} - V_t) \overset{(+)}{-} \sqrt{(V_{DD} - V_t)^2 - \frac{\beta_p}{\beta_n}\left(V_{bias} - (V_{DD} - V_t)\right)^2}$$

From the design rules, $\frac{\beta_p}{\beta_n} = \frac{78.7 \cdot W \cdot L}{265 \cdot W \cdot L}$.

For **configuration a**) (V_{bias} = 0V), this yields $V_{outL} = 0.214V$. In this case, the assumption that the p-channel transistor is saturated holds!

For **configuration b**), the assumption holds, as $V_{outL} = 0.0603V$.

For **configuration c**), the n-channel transistor is assumed to operate in the linear region, while the p-channel transistor is saturated:

$$\begin{aligned}
\frac{1}{2}\beta_p \left(V_{gsp} - V_{tp}\right)^2 &= \beta_n \left(V_{gsn} - V_{tn} - \frac{1}{2}V_{dsn}\right) V_{dsn} \Leftrightarrow \\
\frac{1}{2}\beta_p \left(V_{out} - V_{DD} - V_{tp}\right)^2 &= \beta_n \left(V_{DD} - V_{tn} - \frac{1}{2}V_{out}\right) V_{out}
\end{aligned}$$

Inserting the approximation $V_{tn} = -V_{tp} = V_t$:

$$\begin{aligned}
(1 + \frac{\beta_p}{\beta_n})V_{out}^2 &- 2(1 + \frac{\beta_p}{\beta_n})(V_{DD} - V_t)V_{out} + \frac{\beta_p}{\beta_n}(V_{DD} - V_t)^2 = 0 \Leftrightarrow \\
V_{out}^2 &- 2(V_{DD} - V_t)V_{out} + \frac{\beta_p}{\beta_n + \beta_p}(V_{DD} - V_t)^2 = 0 \Leftrightarrow \\
V_{out} &= (V_{DD} - V_t) \cdot \left(1 \overset{(+)}{-} \sqrt{1 - \frac{\beta_p}{\beta_n + \beta_p}} \right) \Leftrightarrow \\
V_{out} &= (V_{DD} - V_t) \cdot \left(1 \overset{(+)}{-} \sqrt{\frac{\beta_n}{\beta_n + \beta_p}} \right)
\end{aligned}$$

Inserting numerical values yields $V_{outL} = 0.162V$. The assumption holds!

The results are listed in the Table below

	V_{outL}	V_{outH}
Configuration a), $V_{gp} = V_{SS}$	0.214 V	1.8 V
Configuration b), $V_{gp} = V_{bias}$	0.0603 V	1.8 V
Configuration c), $V_{gp} = V_{out}$	0.162 V	1.32 V

Problem 10, No printed solution.

Problem 11

The saturation current, I_{ds}, is determined by the expression

$$I_{ds} = \frac{1}{2}\frac{W}{L}\mu_n C_{ox} \left(V_{gs} - V_T\right)^2 \quad (1)$$

The channel current, I_{ds}, is a drift current. The mobile charge is driven with speed ν_y by the electrical field, E_{ds}, over the channel. At any position, y, the channel current is

$$I_{ds} = Q_c W \nu_y = Q_c W \cdot (-\mu_n E_{ds}) = -\mu_n Q_c W \frac{\delta V(y)}{\delta y} \quad (2)$$

The mobile inversion charge density, Q_c, results from applying the electrical field, E_g, over the gate oxide. At any position, Q_c is:

$$E_g = \frac{1}{t_{ox}}(V_{gs} - V_T - V(y)) \qquad \delta(A \cdot Q_c) = -\epsilon_{SiO_2}(W \cdot \delta y)E_g$$

$$Q_c = -C_{ox}(V_{gs} - V_T - V(y)) \tag{3}$$

Combining Equations (1), (2), and (3), the potential distribution, $V(y)$, is

$$\mu_n C_{ox} W (V_{gs} - V_T - V(y)) \frac{\delta V(y)}{\delta y} = \frac{1}{2}\frac{W}{L}\mu_n C_{ox}(V_{gs} - V_T)^2$$

The potential distribution is determined by integration:

$$\begin{aligned}
\int_0^{V(y)} 2(V_{gs} - V_T - V'(y))\, dV' &= \int_0^y \frac{1}{L}(V_{gs} - V_T)^2\, dy' \Rightarrow \\
2\left((V_{gs} - V_T)V(y) - \frac{1}{2}V^2(y)\right) &= \frac{y}{L}(V_{gs} - V_T)^2 \Rightarrow \\
V(y) = (V_{gs} - V_T) &\cdot \left(1 \overset{(+)}{-} \sqrt{1 - \frac{y}{L}}\right)
\end{aligned} \tag{4}$$

The mobile charge density at any position in the channel is determined by (3) and (4):

$$Q_c = -C_{ox}(V_{gs} - V_T)\sqrt{1 - \frac{y}{L}}$$

and the total mobile channel charge, $Q_{c,\Sigma}$, is

$$\begin{aligned}
Q_{c,\Sigma} &= \int_0^W \int_0^L Q_c dy dz = W \int_0^L -C_{ox}(V_{gs} - V_T)\sqrt{1 - \frac{y}{L}}\, dy \\
&= -WLC_{ox}(V_{gs} - V_T)\left[-\frac{2}{3}(1 - \frac{y}{L})^{\frac{3}{2}}\right]_0^L = -\frac{2}{3}WLC_{ox}(V_{gs} - V_T)
\end{aligned} \tag{5}$$

The required capacitances may now be determined using Equation (5):

$$\begin{aligned}
C_{gs} &\equiv \left.\frac{\delta Q_{g,\Sigma}}{\delta V_{gs}}\right|_{V_{gd}} = -\left.\frac{\delta Q_{c,\Sigma}}{\delta V_{gs}}\right|_{V_{gd}} = \frac{2}{3}WLC_{ox} \\
C_{gd} &\equiv \left.\frac{\delta Q_{g,\Sigma}}{\delta V_{gd}}\right|_{V_{gs}} = -\left.\frac{\delta Q_{c,\Sigma}}{\delta V_{gd}}\right|_{V_{gs}} = 0
\end{aligned}$$

The calculations above assume, that $\frac{\delta Q_D}{\delta V_{gs}} = 0$. Q_D denotes the total depletion charge. This corresponds to Shockley's depletion approximation.
Finally, τ_{tr} in saturation may be calculated using Equations (1) and (5):

$$\tau_{tr} \equiv -\frac{Q_{c,\Sigma}}{I_{ds}} = \frac{4}{3}\frac{L^2}{\mu_n(V_{gs} - V_T)}$$