

A New Foundation for Computing Science
A Research & Experimental Engineering Programme

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Summary

- We argue that computing systems requirements must be based on precisely described domain models.
 - ⋄ We further argue that domain science & engineering offers a new dimension in computing.
 - ⋄ We review our work in this area
 - ⋄ and we hint at a
 - ⊗ research and
 - ⊗ experimental engineering
 - ⋄ programme for
 - ⋄ the first two phases of the triptych of
 - ⊗ domain engineering,
 - ⊗ requirements engineering and
 - ⊗ software design.

Introduction

- This speaker can refer to some substantial evidence that using formal specifications in software development brings some substantial benefits.
 - ❖ Section 2 recalls a first, 1981–1984, instance of such benefits.
 - ❖ Yet, as also outlined in [17, *Bjørner & Havelund: 40 Years of Formal Methods — 10 Obstacles and 3 Possibilities*], “propagation” of formal methods into a wider industry seems lacking.
 - ❖ Although [33, Woodcock et al.] lacks a reference to the formal methods project covered in Sect. 2, it is a fair reference to a number of projects supporting the author’s “benefits” claim.

1.1. The Domain Engineering Claim

- In this talk we wish, however, to not “push” the *formal methods* claim, but to “push” a, or the, *domain science & engineering* claim:
 - ❖ *in order to **design software** one must have a good grasp of its requirements;*
 - ❖ *in order to **prescribe requirements** one must have a good grasp of the underlying domain;*
 - ❖ *so we expect that behind every serious software development there lies a stable **domain description**.*
- This, then, is the purpose of this talk: to “tout” the concept of domain science and engineering, emphasizing, in this talk, the latter.

1.2. Aim of Talk

- So this is neither a *theory* nor a *programming methodology* talk.
 - ❖ It is a review paper:
 - ⊗ “*where do we stand?*” with respect to being able to develop correct software and software that meets customers’ expectations?; and
 - ⊗ “*how can those two issues: ‘correctness’ and ‘meeting expectations’ be improved?*”.

1.3. Structure of Talk

- Section 2 brings two examples:
 - ❖ one of arbitrary, but well-formed transportation nets (illustrated by a road net),
 - ❖ the other of arbitrary, but well-formed pipelines with the flow (laws) of liquid materials.
- The purpose of Sect. 2 is to review a 44 man-year project using formal methods (“lightly”).
 - ❖ We bring this example
 - of a now more than 30 year old project (1981–1984) —
 - ❖ to show an early use of a carefully narrated formal domain description,
 - ❖ a project that we claim to have been a very successful one.

- Section 3 overviews our concept of *TripTych* development:
 - ❖ from domain descriptions,
 - ❖ via requirements prescriptions,
 - ❖ to software design.

We emphasize the domain science & engineering aspects.

- Section 4 Discusses our claim that this *TripTych* suggests a new foundation for computing science.
- Section 5 very briefly draws, what we see as, the necessary conclusions.

A Background Development

- We sketch the structure
 - ❖ of a successful
 - ❖ 44 man year project
 - ❖ which developed a commercial compiler
 - ❖ according to the *TripTych* approach
 - ❖ and using formal specifications —
 - ❖ with success measured in terms of
 - ⊗ meeting customers' expectations
 - ⊗ and being correct.

2.1. The 1981–1984 DDC Ada Compiler Development Project

- In the spring semester (6 months) of 1980 five MSc students
 - ❖ worked out their MSc theses:
 - ❖ **A Formal Description of Ada.**
 - ❖ The four theses were published as [21].
 - ❖ That work became the basis for a full-scale industry-size project:
 - ⊗ *The DDC¹ ADA Compiler Project,*
 - ⊗ funded, in part by the *CEC,*
the *Commission of the European Countries.*
- The project was carried out
 - ❖ according to abstraction and refinement principles laid down in [2], and
 - ❖ can be diagrammed as shown in Fig. 1 on the following slide

¹DDC: Dansk Datamatik Center was an industry-operated R&D centre, 1979–1989.

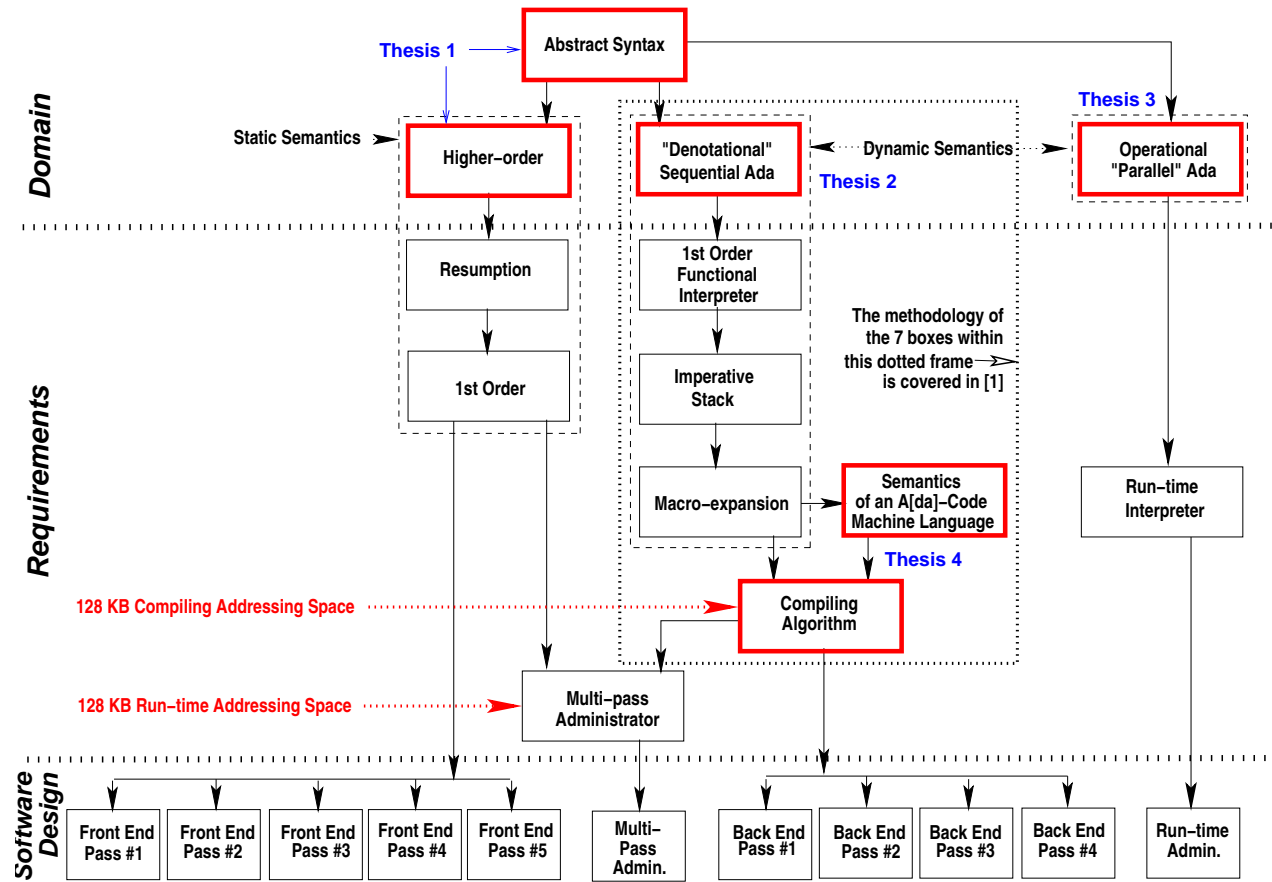


Figure 1: The Ada Compiler Software Development Graph.

- We explain the approach taken to develop, using formal specifications, a compiler for the *Ada* programming language.
- We do so using Fig. 1 as a reference point.
 - ❖ Each box
 - ⊗ represents a specification
 - ⊗ and denotes a mathematical object.
 - ❖ Each directed line between boxes
 - ⊗ represents a step of development,
 - ⊗ and denotes a proof
(of correctness, also a mathematical object).

- There were three phases of development:
 - ❖ the *domain* engineering phase,
 - ❖ the *requirements engineering* phase, and
 - ❖ the *software design* phase.

They are clearly marked in Fig. 1.

- First a formal description was developed for Ada.
 - ❖ This phase is referred to as the ‘Domain’.
 - ❖ It had four stages:
 - ⊗ first the *Abstract Syntax*,
 - ❖ then (developed “concurrently”)
 - ⊗ the *Higher-order Static Semantics*,
 - ⊗ the “*Denotational*” *Dynamic Sequential Semantics* and
 - ⊗ the “*Operational*” *Dynamic Parallel Semantics*.

- Then a phase, *Requirements*, consisting of several stages.
 - ❖ The refinement work represented by each of the boxes,
 - ❖ were conditioned by various requirements.
 - ❖ But we show such only for two boxes: dashed, labelled pointed **red** lines.

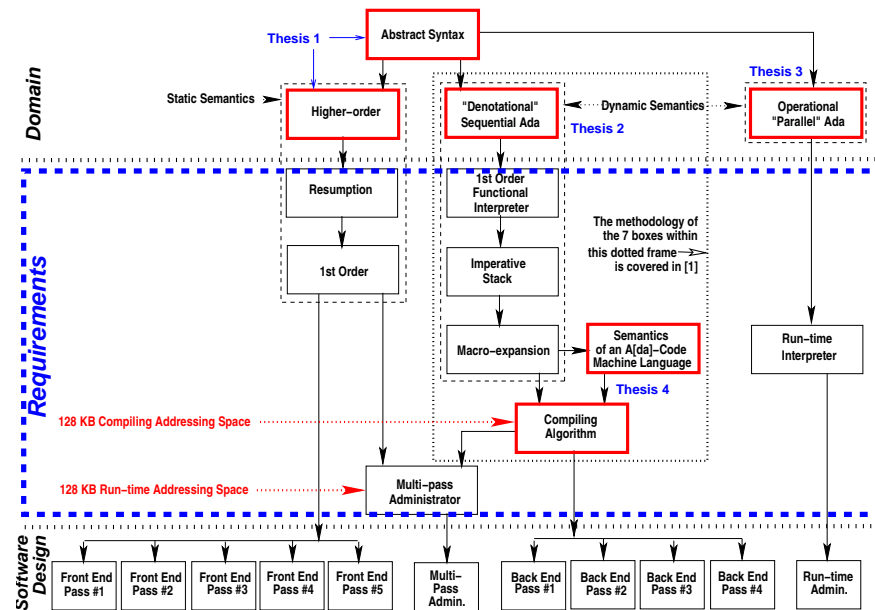


Figure 2: The Ada Compiler Software Development Graph

- ❖ The *Higher-order Static Semantics* is refined in two stages:
 - ⊗ first a *Resumption Static Semantics*
 - ⊗ and then a *First-order Static Semantics*.
- ❖ The “*Denotational*” *Dynamic Sequential Semantics* was refined in three stages:
 - ⊗ a *1st-order Functional Interpreter*,
 - ⊗ a *Imperative Stack* dynamic semantics and
 - ⊗ a *Macro-expansion* dynamic semantics.
- ❖ From the *Operational “Parallel” Semantics* was developed
 - ⊗ an operational *Run-time Semantics*
 - ⊗ for the concurrency constructs of Ada.

- ❖ From the *Macro-expansion* semantics was developed
 - ⊗ the design of an *A[da] Code Language*
 - ⊗ which was given a semantics
 - ⊗ commensurate with the specification language and *Macro-expansion* semantics.
- ❖ And from the
 - ⊗ *Macro-expansion* semantics and the
 - ⊗ *A Code Language*
 - ⊗ was developed a *Compiling Algorithm*
 - * which to every construct of Ada
 - * prescribed a sequence of *A Code*.
- ❖ The *Run-time Interpreter*
 - ⊗ was developed from the
 - ⊗ *Operational “Parallel” Ada Semantics*.

- ❖ Two *requirements assumptions* were:
 - ⊗ the compiler should execute within a 128 KB addressing space, and
 - ⊗ the compiled code should likewise execute within a 128 KB addressing space.
- ❖ Therefore the compiler need be decomposed into a number of passes where a pass was defined as that of a linear reading of of the Ada program text either left-to-right, or right-to-left, and in either pre-, in- or post-order.

- ❖ From the combined
 - ⊗ *1st-order Static Semantics* and the
 - ⊗ *Compiling Algorithm*was, after careful analysis of these, developed
 - ⊗ a specification for a multi-pass administrator.
- The multi-pass analysis and synthesis resulted in
 - ❖ five passes for the statics checks (i.e., “front-end”),
 - ❖ and four passes for the code generator (i.e., “back-end”).
- These concluded the domain and requirements phases
 - ❖ which were all specified in VDM [18, 29]
 - ❖ for a total of approximately
 - ⊗ 10.000, respectively
 - ⊗ 56,000 linesof VDM and formula annotations.

- The

- ❖ nine compiler *Passes*,
- ❖ *Multi-pass Administrator*, and the
- ❖ *Run-time Administrator*

were all coded from their specifications

- ❖ in a subset of the Ada language
- ❖ for which a compiler was developed
- ❖ in parallel with the full-Ada development !

2.2. A Review

2.2.1. Resources

- The above project took place more than 30 years ago!
- Approximately the following man-power resources were used:
 - ❖ For the *Domain* phase: seven people, one year;
 - ❖ for the *Requirements* phase
(exclusive of the *Multi-pass Administrator*:
eleven people, one year;
 - ❖ for the *Multi-pass Administrator*: six people, half a year; and
 - ❖ for the rest (nine *Passes* and the *Administrators*): 12 people, 14 months.
 - ❖ The subset Ada compiler development consumed seven man years.
 - ❖ Thus a total of
 - ⊗ 42 man years was spent on effective development and its management,
 - ⊗ 2 man years on management of donors, funding and marketing.

2.2.2. Formal Methods “Lite”

- VDM was the prime “carrier” of the Ada compiler development.
 - ❖ The domain and the requirements phases were specified in VDM.
 - ❖ No properties of these specifications were formalised let alone proved.
 - ❖ The first 10 years of use by industry on three continents (China, Japan, USA and Europe) revealed few, and only trivial errors:
 - ⊗ less than 2% of original development resources
 - ⊗ were spent on error corrections
 - ⊗ with average “repair” times being in the order of 1–2 days.

2.2.3. Epilogue

- The above-outlined Ada compiler development project was reported in [22, 31].
- The use of formal methods was clear.
- But ‘formal methods’ were not used in any other sense than formal specifications.
- Properties of and relationships between stage specifications were not formalised.
- And yet, the project must be judged an unqualified success for formal methods.
 - ❖ It took far fewer manpower resources than any other Ada compiler development project in those days.
 - ❖ It had far, far fewer “bugs” than any comparable software development project in those days or since.
 - ❖ Yet there were no tools available:
 - ⊗ No VDM syntax checker,
 - ⊗ No specification analyser.
 - ⊗ No nothing!

The Triptych of Software Engineering

- We suggest a **Triptych** view of software engineering:
 - ❖ *before software can be designed and coded*
 - ❖ *we must have a reasonable grasp of “its” requirements; and*
 - ❖ *before requirements can be prescribed*
 - ❖ *we must have a reasonable grasp of “the underlying” domain.*
- To us, therefore, software engineering contains the three sub-disciplines:
 - ❖ domain engineering,
 - ❖ requirements engineering and
 - ❖ software design.

3.1. What's New ?

- So “*What's New ?*” in this ?
 - ⋄ Well, as far as the surveyed compiler development is concerned, nothing:
 - ⊗ that is how one should develop compilers —
 - ⊗ although it seems that it was done only once!

- What can we learn from the example of Sect. 2?
 - ❖ We can postulate that when
 - ❖ there is a formal understanding of the domain —
 - ❖ and of the stages
 - ⊗ from domain to requirements
 - ⊗ and on to software design,
 - ❖ then software can be developed with greater assurance
 - ❖ of meeting users' expectations and be correct
 - ❖ than if not!

- So that is what we are therefore proposing:
 - ❖ to treat the domain,
 - ❖ the application area for software development,
 - ❖ as “a language” whose terms
 - ❖ designate phenomena in the domain
 - ❖ and “spoken/uttered” about by practitioners in the domain.
- So we consider a domain description
 - ❖ to be the description of
 - ⊗ the syntax and
 - ⊗ the semantics
 - ❖ of a language.

3.2. Domain Science & Engineering

3.2.1. What is a Domain ?

- A **domain** is a
 - ⋄ human- and
 - ⋄ artifact-assisted
 - ⋄ arrangement of
 - ⊗ *endurant*, that is spatially “stable”, and
 - ⊗ *perdurant*, that is temporally “fleeting”
- entities.

3.2.2. Example Domains

- To help understand the above delineation
 - ❖ of the ‘domain’ concept
 - ❖ we list some examples
 - ❖ for which we can also refer to some
 - ⊗ either published
 - ⊗ or reported
- domain descriptions:

Example 1 . Manifest Domain Names: Examples of suggestive names of manifest domains are:

- *air traffic,*
- *banks,*
- *container lines,*
- *documents,*
- *hospitals,*
- *manufacturing,*
- *pipelines,*
- *railways and*
- *road nets.*

3.2.3. Comparison to Other Sciences and Their Engineering

- We focus on the natural sciences and their engineerings:
 - ❖ civil (or construction) engineering,
 - ❖ mechanical engineering,
 - ❖ chemical engineering,
 - ❖ electrical, electronics engineering and radio engineering.
- For all of these related technologies engineers
 - ❖ are properly educated,
 - ❖ knows the underlying sciences,
 - ❖ that is, the domains of their artifacts.
- Not so, today, 2015, for software engineers for the domains listed in Example 1 on the previous slide.

- Software engineers asked to develop software for either of
 - ❖ air traffic control, ❖ container lines, ❖ railways,
 - ❖ banking. ❖ health care, ❖ road pricing,
- etcetera, are expected to find out, themselves,
- ❖ what the relevant domain is,
 - ❖ how it behaves, etc.
 - ❖ No wonder that it often fails!

3.2.4. Domain Descriptions: Internet References

- Now, we would not postulate the above without firm evidence.
- “*Proof in the pudding*” sort-of-evidence that domains can indeed be properly, informally and formally described.
- We shall first mention some existing descriptions before we exemplify fragments of such descriptions.

1 *A Railway Systems Domain*

D.Bjørner et al.

- Scheduling and Rescheduling of Trains; C.W.George and S.Prehn, 1996, [amore/scheduling.pdf](#)
- Formal Software Techniques in Railway Systems; 2000, [amore/dines-fac.pdf](#)
- Dynamics of Railway Systems; 2000, [amore/ifac-dynamics.pdf](#)
- Railway Staff Rostering; A.Strupchanska et al., 2003, [amore/albena-amore.pdf](#)
- Train Maintenance Routing; M.Peñicka et al., 2003, [amore/martin-amore.pdf](#)
- Train Composition and Decomposition: Domain and Requirements (draft), P.Karras et al., 2003, [amore/panos-amore.pdf](#)

- 2 *Models of IT Security. Security Rules & Regulations*, [it-security.pdf](#), 2006.
See [13]. A sketch is given of the IT security rules laid down by ISO
- 3 *A Container Line Industry Domain*, [container-paper.pdf](#), 2007
- 4 *The “Market”: Consumers, Retailers, Wholesalers, Producers*, [themarket.pdf](#), 2007 See [3].
- 5 *What is Logistics ?* [logistics.pdf](#), 2009
- 6 *A Domain Model of Oil Pipelines*, [pipeline.pdf](#), 2009
- 7 *Transport Systems*, [comet/comet1.pdf](#), 2010
- 8 *The Tokyo Stock Exchange*, [todai/tse-1.pdf](#) and [todai/tse-2.pdf](#), 2010
- 9 *On Development of Web-based Software. A Divertimento*, [wdfftp.pdf](#), 2010
- 10 *Documents (incomplete draft)*, [doc-p.pdf](#), See [12]. 2013

3.2.5. An Example: Road Nets, Vehicles and Traffic

3.2.5.1 Parts

- The root domain, $\Delta_{\mathcal{D}}$,
- the step-wise unfolding of whose description is to be exemplified, is that of a **composite traffic system**
 - ◇ with a road net,
 - ◇ with a fleet of vehicles and
 - ◇ of whose individual position on the road net we can speak, that is, monitor.

1 We analyse the composite traffic system into

- a. a composite road net,
- b. a composite fleet (of vehicles), and

type

- 1. Δ_{Δ}
- 1a.. N_{Δ}
- 1b.. F_{Δ}
- 1c.. M_{Δ}

value

- 1a.. **obs_part** $_N_{\Delta}: \Delta_{\Delta} \rightarrow N_{\Delta}$
- 1b.. **obs_part** $_F_{\Delta}: \Delta_{\Delta} \rightarrow F_{\Delta}$

c. an atomic monitor.

2 The road net consists of two composite parts,

- a. an aggregation of hubs and
- b. an aggregation of links.

1c.. **obs_part** $_M_{\Delta}: \Delta_{\Delta} \rightarrow M_{\Delta}$

type

- 2a.. HA_{Δ}
- 2b.. LA_{Δ}

value

- 2a.. **obs_part** $_HA_{\Delta}: N_{\Delta} \rightarrow HA_{\Delta}$
- 2b.. **obs_part** $_LA_{\Delta}: N_{\Delta} \rightarrow LA_{\Delta}$

3 Hub aggregates are sets of hubs.

4 Link aggregates are sets of links.

5 Fleets are sets of vehicles.

6 We introduce some auxiliary

type

3. $H_{\Delta}, HS_{\Delta} = H_{\Delta}\text{-set}$

4. $L_{\Delta}, LS_{\Delta} = L_{\Delta}\text{-set}$

5. $V_{\Delta}, VS_{\Delta} = V_{\Delta}\text{-set}$

value

3. $\text{obs_part_HS}_{\Delta}: HA_{\Delta} \rightarrow HS_{\Delta}$

functions.

a. **links** extracts the links of a network.

b. **hubs** extracts the hubs of a network.

4. $\text{obs_part_LS}_{\Delta}: LA_{\Delta} \rightarrow LS_{\Delta}$

5. $\text{obs_part_VS}_{\Delta}: F_{\Delta} \rightarrow VS_{\Delta}$

6a.. $\text{links}_{\Delta}: \Delta_{\Delta} \rightarrow L\text{-set}$

6a.. $\text{links}_{\Delta}(\delta_{\Delta}) \equiv \text{obs_part_LS}(\text{obs_part_LA}(\delta_{\Delta}))$

6b.. $\text{hubs}_{\Delta}: \Delta_{\Delta} \rightarrow H\text{-set}$

6b.. $\text{hubs}_{\Delta}(\delta_{\Delta}) \equiv \text{obs_part_HS}(\text{obs_part_HA}(\delta_{\Delta}))$

3.2.5.2 Unique Identifiers

We cover the unique identifiers of all parts, whether needed or not.

7 Nets, hub and link aggregates, hubs and links, fleets, vehicles and the monitor all

- a. have unique identifiers
- b. such that all such are distinct, and
- c. with corresponding observers.

8 We introduce some auxiliary functions:

- a. **xtr_lis** extracts all link

identifiers of a traffic system.

- b. **xtr_his** extracts all hub identifiers of a traffic system.
- c. given an appropriate link identifier and a net **get_link** ‘retrieves’ the designated link.
- d. given an appropriate hub identifier and a net **get_hub** ‘retrieves’ the designated hub.

type

7a.. NI, HAI, LAI, HI, LI, FI, VI, MI

value

7c.. **uid_NI**: $N_{\Delta} \rightarrow NI$

7c.. **uid_HAI**: $HA_{\Delta} \rightarrow HAI$

7c.. **uid_LAI**: $LA_{\Delta} \rightarrow LAI$

7c.. **uid_HI**: $H_{\Delta} \rightarrow HI$

7c.. **uid_LI**: $L_{\Delta} \rightarrow LI$

7c.. **uid_FI**: $F_{\Delta} \rightarrow FI$

7c.. **uid_VI**: $V_{\Delta} \rightarrow VI$

7c.. **uid_MI**: $M_{\Delta} \rightarrow MI$

axiom

7b.. $NI \cap HAI = \emptyset, NI \cap LAI = \emptyset, NI \cap HI = \emptyset, \text{ etc.}$

where axiom 7b. is expressed semi-formally, in mathematics.

value8a.. $xtr_lis: \Delta_{\Delta} \rightarrow \underline{LI\text{-}set}$ 8a.. $xtr_lis(\delta_{\Delta}) \equiv$ 8a.. **let** $ls = links(\delta_{\Delta})$ **in** $\{uid_LI(l) \mid l:L \cdot l \in ls\}$ **end**8b.. $xtr_his: \Delta_{\Delta} \rightarrow \underline{HI\text{-}set}$ 8b.. $xtr_his(\delta_{\Delta}) \equiv$ 8b.. **let** $hs = hubs(\delta_{\Delta})$ **in** $\{uid_HI(h) \mid h:H \cdot k \in hs\}$ **end**8c.. $get_link: LI \rightarrow \Delta_{\Delta} \xrightarrow{\sim} L$ 8c.. $get_link(li)(\delta_{\Delta}) \equiv$ 8c.. **let** $ls = links(\delta_{\Delta})$ **in**8c.. **let** $l:L \cdot l \in ls \wedge li=uid_LI(l)$ **in** l **end end**8c.. **pre:** $li \in xtr_lis(\delta_{\Delta})$ 8d.. $get_hub: HI \rightarrow \Delta_{\Delta} \xrightarrow{\sim} H$ 8d.. $get_hub(hi)(\delta_{\Delta}) \equiv$ 8d.. **let** $hs = hubs(\delta_{\Delta})$ **in**8d.. **let** $h:H \cdot h \in hs \wedge hi=uid_HI(h)$ **in** h **end end**8d.. **pre:** $hi \in xtr_his(\delta_{\Delta})$

3.2.5.3 Mereology

- 9 Links are connected to exactly two distinct hubs.
- 10 Hubs are connected to zero or more links.
- 11 For a given net the link and hub identifiers of the mereology of hubs and links must be those of links and hubs, respectively, of the net.

type

9. $LM' = \underline{HI\text{-set}}, LM = \{ | \text{his:} \underline{HI\text{-set}} \cdot \underline{\text{card}}(\text{his})=2 | \}$

10. $HM = \underline{LI\text{-set}}$

value

9. $\text{mereo_L}: L \rightarrow LM$

10. $\text{mereo_H}: H \rightarrow HM$

axiom [Well-formedness of Road Nets, N]

11. $\forall n:N, l:L, h:H. l \in \text{obs_part_Ls}(\text{obs_part_LC}(n)) \wedge h \in \text{obs_part_Hs}(\text{obs_part_HC}(n))$

11. $\underline{\text{let}} \text{his} = \text{mereology_H}(l), \text{lis} = \text{mereology_H}(h) \underline{\text{in}}$

11. $\text{his} \subseteq U \{ \text{uid_H}(h) \mid h \in \text{obs_part_Hs}(\text{obs_part_HC}(n)) \}$

11. $\wedge \text{lis} \subseteq U \{ \text{uid_H}(l) \mid l \in \text{obs_part_Ls}(\text{obs_part_LC}(n)) \} \underline{\text{end}}$

3.2.5.4 Attributes

We may not have shown all of the attributes mentioned below — so consider them informally introduced!

- **Hubs:**

- ❖ *locations* are considered static,
- ❖ *wear and tear* (condition of road surface) is considered inert,
- ❖ *hub states* and *hub state spaces* are considered programmable;

- **Links:**

- ❖ *lengths* and *locations* are considered static,
- ❖ *wear and tear* (condition of road surface) is considered inert,
- ❖ *link states* and *link state spaces* are considered programmable;

- **Vehicles:**

- ❖ *manufacturer name*, *engine type* (whether diesel, gasoline or electric) and *engine power* (kW/horse power) are considered static;
- ❖ *velocity* and *acceleration* may be considered reactive (i.e., a function of gas pedal position, etc.),
- ❖ *global position* (informed via a GNSS: Global Navigation Satellite System) and *local position* (calculated from a global position) are considered biddable

We treat one attribute each for hubs, links, vehicles and the monitor. First we treat hubs.

12 Hubs

- a. have *hub states* which are sets of pairs of identifiers of links connected to the hub²,
- b. and have *hub state spaces* which are sets of hub states³.

13 For every net,

- a. link identifiers of a hub state must designate links of that net.
- b. Every hub state of a net must be in the hub state space of that hub.

14 Hubs have geodetic and cadastral location.

15 We introduce an auxiliary function: `xtr_lis` extracts all link identifiers of a hub state.

²A hub state “signals” which input-to-output link connections are open for traffic.

³A hub state space indicates which hub states a hub may attain over time.

type

12a.. $H\Sigma = (LI \times LI)\text{-set}$

12b.. $H\Omega = H\Sigma\text{-set}$

value

12a.. $\text{attr_H}\Sigma: H \rightarrow H\Sigma$

12b.. $\text{attr_H}\Omega: H \rightarrow H\Omega$

axiom

13. $\forall \delta: \Delta,$

13. **let** $hs = \text{hubs}(\delta)$ **in**

13. $\forall h: H \cdot h \in hs \cdot$

13a.. $\text{xtr_lis}(h) \subseteq \text{xtr_lis}(\delta)$

13b.. $\wedge \text{attr_}\Sigma(h) \in \text{attr_}\Omega(h)$

13. **end**

type

14. HGCL

value

14. $\text{attr_HGCL}: H \rightarrow \text{HGCL}$

15. $\text{xtr_lis}: H \rightarrow LI\text{-set}$

15. $\text{xtr_lis}(h) \equiv$

15. $\{li \mid li: LI, (li', li''): LI \times LI \cdot$

15. $(li', li'') \in \text{attr_H}\Sigma(h) \wedge li \in \{li', li''\}\}$

Then links.

16 Links have lengths.

17 Links have geodetic and cadastral location.

18 Links have states and state spaces:

- a. States modeled here as pairs, (hi', hi'') , of identifiers the hubs with which the links are connected and indicating directions (from hub h' to hub h'' .) A link state can thus have 0, 1, 2, 3 or 4 such pairs.
- b. State spaces are the set of all the link states that a link may enjoy.

type

- 16. LEN
- 17. LGCL
- 18a.. $L\Sigma = (HI \times HI)\text{-set}$
- 18b.. $L\Omega = L\Sigma\text{-set}$

value

- 16. **attr_LEN**: $L \rightarrow \text{LEN}$
- 17. **attr_LGCL**: $L \rightarrow \text{LGCL}$
- 18a.. **attr_LΣ**: $L \rightarrow L\Sigma$
- 18b.. **attr_LΩ**: $L \rightarrow L\Omega$

axiom

- 18. $\forall n:N \cdot$
- 18. **let** $ls = \text{xtr_links}(n)$, $hs = \text{xtr_hubs}(n)$ **in**
- 18. $\forall l:L \cdot l \in ls \Rightarrow$
- 18a.. **let** $l\sigma = \text{attr_L}\Sigma(l)$ **in**
- 18a.. $0 \leq \text{card } l\sigma \leq 4$
- 18a.. $\wedge \forall (hi', hi''):(HI \times HI) \cdot (hi', hi'') \in l\sigma \Rightarrow$
- 18a.. $\{\text{get_H}(hi')(n), \text{get_H}(hi'')(n)\} = \text{mereo_L}(l)$
- 18b.. $\wedge \text{attr_L}\Sigma(l) \in \text{attr_L}\Omega(l)$
- 18. **end end**

Then vehicles.

19 Every vehicle of a traffic system has a position which is either ‘on a link’ or ‘at a hub’.

- a. An ‘on a link’ position has four elements: a unique link identifier which must designate a link of that traffic system and a pair of unique hub identifiers which must be those of the mereology of that link.
- b. The ‘on a link’ position real is the fraction, thus properly between 0 (zero) and 1 (one) of the length from the first identified hub “down the link” to the second identifier hub.
- c. An ‘at a hub’ position has three elements: a unique hub identifier and a pair of unique link identifiers — which must be in the hub state.

type

- 19. VPos = onL | atH
- 19a.. onL :: LI HI HI R
- 19b.. R = **Real** **axiom** $\forall r:R . 0 \leq r \leq 1$
- 19c.. atH :: HI LI LI

value

- 19. attr_VPos: $V_{\Delta} \rightarrow VPos$

axiom

- 19a.. $\forall n_{\Delta}:N_{\Delta}, onL(li,fhi,thi,r):VPos .$
- 19a.. $\exists l_{\Delta}:L_{\Delta} \cdot l_{\Delta} \in \mathbf{obs_part_LS}(\mathbf{obs_part_N}_{\Delta}(n_{\Delta}))$
- 19a.. $\Rightarrow li = \mathbf{uid_L}_{\Delta}(l) \wedge \{fhi,thi\} = \mathbf{mereo_L}_{\Delta}(l_{\Delta}),$
- 19c.. $\forall n_{\Delta}:N_{\Delta}, atH(hi,fli,tli):VPos .$
- 19c.. $\exists h_{\Delta}:H_{\Delta} \cdot h_{\Delta} \in \mathbf{obs_part_HS}_{\Delta}(\mathbf{obs_part_N}(n_{\Delta}))$
- 19c.. $\Rightarrow hi = \mathbf{uid_H}_{\Delta}(h_{\Delta}) \wedge (fli,tli) \in \mathbf{attr_L\Sigma}(h_{\Delta})$

And finally monitors. We consider only one monitor attribute.

20 The monitor has a vehicle traffic attribute.

- a. For every vehicle of the road transport system the vehicle traffic attribute records a possibly empty list of time marked vehicle positions.
- b. These vehicle positions are alternate sequences of ‘on link’ and ‘at hub’ positions
 - i such that any sub-sequence of ‘on link’ positions record the same link identifier, the same pair of ‘to’ and ‘from’ hub identifiers and increasing fractions,
 - ii such that any sub-segment of ‘at hub’ positions are identical,
 - iii such that vehicle transition from a link to a hub is commensurate with the link and hub mereologies, and
 - iv such that vehicle transition from a hub to a link is commensurate with the hub and link mereologies.

type

20. Traffic = VI \mapsto (T \times VPos)*

value

20. attr_Traffic: M \rightarrow Traffic

axiom

20b.. $\forall \delta: \Delta \bullet$

20b.. **let** m = **obs_part**_{M Δ} (δ) **in**

20b.. **let** tf = **attr_Traffic**(m) **in**

20b.. **dom** tf \subseteq xtr_vis(δ) \wedge

20b.. $\forall vi:VI \bullet vi \in$ **dom** tf \bullet

20b.. **let** tr = tf(vi) **in**

20b.. $\forall i,i+1: \mathbf{Nat} \bullet \{i,i+1\} \subseteq$ **dom** tr \bullet

20b.. **let** (t,vp)=tr(i),(t',vp')=tr(i+1) **in**

20b.. t < t'

20(b.)i. \wedge **case** (vp,vp') **of**

20(b.)i. (onL(li,fhi,thi,r),onL(li',fhi',thi',r'))

20(b.)i. $\rightarrow li=li' \wedge fhi=fhi' \wedge thi=thi' \wedge r \leq r'$

20(b.)i. $\wedge li \in$ xtr_lis(δ)

20(b.)i. $\wedge \{fhi,thi\} =$ mereo_L(get_link(li)(δ),

20(b.)ii. (atH(hi,fli,tli),atH(hi',fli',tli'))

20(b.)ii. $\rightarrow hi=hi' \wedge fli=fli' \wedge tli=tli'$

20(b.)ii. $\wedge hi \in$ xtr_his(δ)

20(b.)ii. $\wedge (fli,tli) \in$ mereo_H(get_hub(hi)(δ),

20(b.)iii. (onL(li,fhi,thi,1),atH(hi,fli,tli))

20(b.)iii. $\rightarrow li=fli \wedge thi=hi$

20(b.)iii. $\wedge \{li,tli\} \subseteq$ xtr_lis(δ)

20(b.)iii. $\wedge \{fhi,thi\} =$ mereo_L(get_link(li)(δ))

20(b.)iii. $\wedge hi \in$ xtr_his(δ)

20(b.)iii. $\wedge (fli,tli) \in$ mereo_H(get_hub(hi)(δ),

20(b.)iv. (atH(hi,fli,tli),onL(li',fhi',thi',0))

20(b.)iv. \rightarrow etcetera,

20b.. $_ \rightarrow$ **false**

20b.. **end end end end end**

3.2.6. Another Example: Pipelines

3.2.6.1 Parts

21 A pipeline consists of an indefinite number of pipeline units.

22 A pipeline units is either a well, or a pipe, or a pump, or a valve, or a fork, or a join, or a sink.

23 All these unit sorts are atomic and disjoint.

type

21. PL, U, We, Pi, Pu, Va, Fo, Jo, Si

21. Well, Pipe, Pump, Valv, Fork, Join, Sink

value

21. **obs_part_Us**: PL \rightarrow U-set

type

22. $U == We \mid Pi \mid Pu \mid Va \mid Fo \mid Jo \mid Si$

23. We::Well, Pi::Pipe, Pu::Pump, Va::Valv, Fo:Fork, Jo::Join, Si::Sink

3.2.6.2 Unique Identifiers

24 Every pipeline unit has a unique identifier.

type

24. UI

value

24. **uid**_U: $U \rightarrow UI$

3.2.6.3 Materials

- 25 Applying `obs_material_sorts_U` to any pipeline unit, $u:U$, yields
- a type clause stating the material sort **LoG** for some further undefined liquid or gaseous material, and
 - a material observer function signature.

type

25a. `LoG`

value

25b. `obs_mat_LoG: U → LoG`

3.2.6.4 Mereology

- Pipeline units serve to conduct fluid or gaseous material.
- The flow of these occur in only one direction: from so-called input to so-called output.

26 Wells have exactly one connection to an output unit.

27 Pipes, pumps and valves have exactly one connection from an input unit and one connection to an output unit.

28 Forks have exactly one connection from an input unit and exactly two connections to distinct output units.

29 Joins have exactly one two connection from distinct input units and one connection to an output unit.

30 Sinks have exactly one connection from an input unit.

31 Thus we model the mereology of a pipeline unit as a pair of disjoint sets of unique pipeline unit identifiers.

type31. $UM' = (UI_set \times UI_set)$ 31. $UM = \{ |(iuis, ouis): UI_set \times UI_set \cdot iuis \cap ouis = \{ \} | \}$ **value**31. **mereo_U**: UM**axiom** [Well-formedness of Pipeline Systems, PLS (0)] $\forall pl: PL, u: U \cdot u \in \mathbf{obs_part_Us}(pl) \Rightarrow$ **let** (iuis, ouis) = mereo_U(u) **in****case** (**card** iuis, **card** ouis) **of**26. $(0, 1) \rightarrow \mathbf{is_We}(u),$ 27. $(1, 1) \rightarrow \mathbf{is_Pi}(u) \vee \mathbf{is_Pu}(u) \vee \mathbf{is_Va}(u),$ 28. $(1, 2) \rightarrow \mathbf{is_Fo}(u),$ 29. $(2, 1) \rightarrow \mathbf{is_Jo}(u),$ 30. $(1, 0) \rightarrow \mathbf{is_Si}(u)$ **end end**

3.2.6.5 Attributes

- Let us postulate a [n attribute] sort **Flow**.
- We now wish to examine the flow of liquid (or gaseous) material in pipeline units.
- We use two types
 - 32 **F** for “productive” flow, and **L** for wasteful leak.
- Flow and leak is measured, for example, in terms of volume of material per second.
- We then postulate the following unit attributes
 - ❖ “measured” at the point of in- or out-flow
 - ❖ or in the interior of a unit.

- 33 current flow of material into a unit input connector,
- 34 maximum flow of material into a unit input connector while maintaining laminar flow,
- 35 current flow of material out of a unit output connector,
- 36 maximum flow of material out of a unit output connector while maintaining laminar flow,
- 37 current leak of material at a unit input connector,
- 38 maximum guaranteed leak of material at a unit input connector,
- 39 current leak of material at a unit input connector,
- 40 maximum guaranteed leak of material at a unit input connector,
- 41 current leak of material from “within” a unit, and
- 42 maximum guaranteed leak of material from “within” a unit.

type

32. F, L

value33. **attr_cur_iF**: $U \rightarrow UI \rightarrow F$ 34. **attr_max_iF**: $U \rightarrow UI \rightarrow F$ 35. **attr_cur_oF**: $U \rightarrow UI \rightarrow F$ 36. **attr_max_oF**: $U \rightarrow UI \rightarrow F$ 37. **attr_cur_iL**: $U \rightarrow UI \rightarrow L$ 38. **attr_max_iL**: $U \rightarrow UI \rightarrow L$ 39. **attr_cur_oL**: $U \rightarrow UI \rightarrow L$ 40. **attr_max_oL**: $U \rightarrow UI \rightarrow L$ 41. **attr_cur_L**: $U \rightarrow L$ 42. **attr_max_L**: $U \rightarrow L$

- The maximum flow attributes are static attributes and are typically provided by the manufacturer as indicators of flows below which laminar flow can be expected.
- The current flow attributes are dynamic attributes

3.2.6.6 Intra Unit Flow and Leak Law

43 For every unit of a pipeline system, except the well and the sink units, the following law apply.

44 The flows into a unit equal

- a. the leak at the inputs
- b. plus the leak within the unit
- c. plus the flows out of the unit
- d. plus the leaks at the outputs.

axiom [Well–formedness of Pipeline Systems, PLS (1)]

43. $\forall pls:PLS, b:B \setminus We \setminus Si, u:U \cdot$

43. $b \in \mathbf{obs_part_Bs}(pls) \wedge u = \mathbf{obs_part_U}(b) \Rightarrow$

43. **let** (iuis,ouis) = **mereo_U**(u) **in**

44. $\mathbf{sum_cur_iF}(iuis)(u) =$

44a.. $\mathbf{sum_cur_iL}(iuis)(u)$

44b.. $\oplus \mathbf{attr_cur_L}(u)$

44c.. $\oplus \mathbf{sum_cur_oF}(ouis)(u)$

44d.. $\oplus \mathbf{sum_cur_oL}(ouis)(u)$

43. **end**

- 45 The **sum_cur_iF** (cf. Item 44) sums current input flows over all input connectors.
- 46 The **sum_cur_iL** (cf. Item 44a.) sums current input leaks over all input connectors.
- 47 The **sum_cur_oF** (cf. Item 44c.) sums current output flows over all output connectors.
- 48 The **sum_cur_oL** (cf. Item 44d.) sums current output leaks over all output connectors.

$$45. \quad \text{sum_cur_iF}: \underline{\text{UI-set}} \rightarrow \text{U} \rightarrow \text{F}$$

$$45. \quad \text{sum_cur_iF}(iuis)(u) \equiv \bigoplus \{ \text{attr_cur_iF}(ui)(u) \mid ui: \text{UI} \cdot ui \in iuis \}$$

$$46. \quad \text{sum_cur_iL}: \underline{\text{UI-set}} \rightarrow \text{U} \rightarrow \text{L}$$

$$46. \quad \text{sum_cur_iL}(iuis)(u) \equiv \bigoplus \{ \text{attr_cur_iL}(ui)(u) \mid ui: \text{UI} \cdot ui \in iuis \}$$

$$47. \quad \text{sum_cur_oF}: \underline{\text{UI-set}} \rightarrow \text{U} \rightarrow \text{F}$$

$$47. \quad \text{sum_cur_oF}(ouis)(u) \equiv \bigoplus \{ \text{attr_cur_iF}(ui)(u) \mid ui: \text{UI} \cdot ui \in ouis \}$$

$$48. \quad \text{sum_cur_oL}: \underline{\text{UI-set}} \rightarrow \text{U} \rightarrow \text{L}$$

$$48. \quad \text{sum_cur_oL}(ouis)(u) \equiv \bigoplus \{ \text{attr_cur_iL}(ui)(u) \mid ui: \text{UI} \cdot ui \in ouis \}$$

$$\bigoplus: (\text{F}|\text{L}) \times (\text{F}|\text{L}) \rightarrow \text{F}$$

3.2.6.7 Inter Unit Flow and Leak Law

49 For every pair of connected units of a pipeline system the following law apply:

- a. the flow out of a unit directed at another unit minus the leak at that output connector
- b. equals the flow into that other unit at the connector from the given unit plus the leak at that connector.

axiom [Well-formedness of Pipeline Systems, PLS (2)]

49. $\forall pls:PLS, b, b':B, u, u':U.$
49. $\{b, b'\} \subseteq \mathbf{obs_part_Bs}(pls) \wedge b \neq b' \wedge u' = \mathbf{obs_part_U}(b')$
49. $\wedge \mathbf{let} (iuis, ouis) = \mathbf{mereo_U}(u), (iuis', ouis') = \mathbf{mereo_U}(u'),$
49. $ui = \mathbf{uid_U}(u), ui' = \mathbf{uid_U}(u') \mathbf{in}$
49. $ui \in iuis \wedge ui' \in ouis' \Rightarrow$
- 49a.. $\mathbf{attr_cur_oF}(u')(ui') - \mathbf{attr_leak_oF}(u')(ui')$
- 49b.. $= \mathbf{attr_cur_iF}(u)(ui) + \mathbf{attr_leak_iF}(u)(ui)$
49. **end**
49. **comment:** b' precedes b

- From the above two laws one can prove the **theorem**:
 - ❖ what is pumped from the wells equals
 - ❖ what is leaked from the systems plus what is output to the sinks.

3.2.7. Domain Descriptions: Methodology

- By a **method** we shall understand
 - ❖ a set of **principles**
 - ❖ for **selecting** and **applying**
 - ❖ **techniques** and
 - ❖ **tools**
 - ❖ for **constructing**
 - ❖ **artifacts**
- By **methodology** we shall understand
 - ❖ the **study** and **knowledge**
 - ❖ of **methods**.

- The tools of the domain description method centers around two kinda of **prompts**.
- By a **prompt** we shall understand something that
 - ❖ induces an action,
 - ❖ an occasion or incitement to inspire, or
 - ❖ an assist suggesting something to be expressed.
- There are two kinds of prompts:
 - ❖ **analysis prompts** and
 - ❖ **description prompts**.

- The analysis prompts to be summarised next
 - ❖ can be thought of as predicates
 - ❖ that the domain engineer applies
 - ❖ to phenomena of the domain
 - ❖ yielding true, false or undefined answers.
- The description prompts to be summarised next
 - ❖ are applied, by the domain engineer,
 - ❖ to phenomena of the domain
 - ❖ for which preceding analysis prompts
 - ❖ has yielded truth answers.
- Thus the *domain analysis & description process*
 - ❖ alternates between analysis prompts
 - ❖ and description prompts.
- The **domain description method** is here specialised to **manifest domains** [10].

First the domain engineer cum scientist examines a perceived domain phenomena, ϕ :

- `is_entity(ϕ)`, and if **true**,
- then inquires which of `is_endurant(ϕ)`
- or `is_perdurant(ϕ)` holds.
- If `is_endurant(ϕ)` holds then the domain analyser inquires as to whether `is_discrete(ϕ)` or `is_continuous(ϕ)` holds.
- If `is_discrete(ϕ)` holds then `is_part(ϕ)` holds,
- otherwise either of `is_material` or `is_component` holds.
- If `is_part(ϕ)` then either `is_atomic(ϕ)` or `is_composite(ϕ)`.
- If `is_composite(ϕ)` holds then `observe_parts(ϕ)` yields some parts that can now be analysed, eventually leading the domain analyser to conclude that the part ϕ can be described.

- By applying **observe_part_sorts(ϕ)** to a composite domain δ we then obtain its constituent parts —
 - ❖ as exemplified in formula lines 1.–1c..

type

1. Δ_{Δ}

1a.. N_{Δ}

1b.. F_{Δ}

1c.. M_{Δ}

value

1a.. **obs_part** N_{Δ} : $\Delta_{\Delta} \rightarrow N_{\Delta}$

1b.. **obs_part** F_{Δ} : $\Delta_{\Delta} \rightarrow F_{\Delta}$

1c.. **obs_part** M_{Δ} : $\Delta_{\Delta} \rightarrow M_{\Delta}$

❖ and similarly formula lines 2a.–2b.

type

2a.. HA_{Δ}

2b.. LA_{Δ}

value

2a.. **obs_part** $_{HA_{\Delta}}$: $N_{\Delta} \rightarrow HA_{\Delta}$

2b.. **obs_part** $_{LA_{\Delta}}$: $N_{\Delta} \rightarrow LA_{\Delta}$

- Some composite parts may be modelled by concrete types:
`has_concrete_type(ϕ)`
- in which case `observe_part_types(ϕ)` will yield those concrete types
 - ❖ as exemplified in formula lines 3.–5

type

3. $H_{\Delta}, HS_{\Delta} = H_{\Delta}\text{-set}$

4. $L_{\Delta}, LS_{\Delta} = L_{\Delta}\text{-set}$

5. $V_{\Delta}, VS_{\Delta} = V_{\Delta}\text{-set}$

value

3. `obs_part_HS Δ` : $HA_{\Delta} \rightarrow HS_{\Delta}$

4. `obs_part_LS Δ` : $LA_{\Delta} \rightarrow LS_{\Delta}$

5. `obs_part_VS Δ` : $F_{\Delta} \rightarrow VS_{\Delta}$

❖ and in formula lines 21

type

21. PL, U, ...

value

21. **obs_part_Us**: PL \rightarrow U-set

- Once the atomic and composite parts of a domain has been settled their properties:
 - ❖ unique identifiers,
 - ❖ mereology and
 - ❖ attributes
- can be analysed and described.

- First their uniqueness: **observe_unique_identifiers**, such as f.ex. illustrated by formula lines 7a.–7b.:

type

7a.. NI, HAI, LAI, HI, LI, FI, VI, MI

value

7c.. **uid_NI**: $N_{\Delta} \rightarrow NI$

7c.. **uid_HAI**: $HA_{\Delta} \rightarrow HAI$, **uid_LAI**: $LA_{\Delta} \rightarrow LAI$

7c.. **uid_HI**: $H_{\Delta} \rightarrow HI$, **uid_LI**: $L_{\Delta} \rightarrow LI$

7c.. **uid_FI**: $F_{\Delta} \rightarrow FI$

7c.. **uid_VI**: $V_{\Delta} \rightarrow VI$

7c.. **uid_MI**: $M_{\Delta} \rightarrow MI$

axiom

7b.. $NI \cap HAI = \emptyset$, $NI \cap LAI = \emptyset$, $NI \cap HI = \emptyset$, etc.

- Once all parts have been identified
one can inquire as to their mereology:
how parts relate to other parts: if $\text{has_mereology}(\phi)$ holds
- then $\text{observe_mereology}(\phi)$ yields which specific other parts,
of same or other sorts, such as for example in formula lines 9.–10

type

9. $\text{LM}' = \text{HI-set}, \text{LM} = \{|\text{his:HI-set} \cdot \text{card}(\text{his})=2|\}$

10. $\text{HM} = \text{LI-set}$

value

9. $\text{mereo_L}: \text{L} \rightarrow \text{LM}$

10. $\text{mereo_H}: \text{H} \rightarrow \text{HM}$

or formula lines 31

type

31. $\text{UM}' = (\text{UI-set} \times \text{UI-set})$

31. $\text{UM} = \{|\text{(iuis,ouis):UI-set} \times \text{UI-set} \cdot \text{iuis} \cap \text{ouis} = \{\}\|\}$

value

31. $\text{mereo_U}: \text{UM}$

- Finally a last set of properties of parts can be investigated, namely their attributes.
 - ❖ Any part, ϕ , may have any number of attributes.
 - ❖ The analysis prompt `attribute_names(ϕ)` yields names of attributes. — with
 - ❖ the description prompt `observe_attributes(ϕ)` yielding their description —
 - ❖ as in formula lines 12a.–12b.

type

$$12a.. \quad H\Sigma = (LI \times LI)\text{-}\underline{\text{set}}$$

$$12b.. \quad H\Omega = H\Sigma\text{-}\underline{\text{set}}$$

value

$$12a.. \quad \text{attr_}H\Sigma: H \rightarrow H\Sigma$$

$$12b.. \quad \text{attr_}H\Omega: H \rightarrow H\Omega$$

❖ or in formula lines 16.–18b..

type

16. LEN

17. LGCL

18a.. $L\Sigma = (HI \times HI)\text{-set}$

18b.. $L\Omega = L\Sigma\text{-set}$

value

16. **attr_LEN**: $L \rightarrow \text{LEN}$

17. **attr_LGCL**: $L \rightarrow \text{LGCL}$

18a.. **attr_LΣ**: $L \rightarrow L\Sigma$

18b.. **attr_LΩ**: $L \rightarrow L\Omega$

- There are other aspects to the methodology
 - ◇ analysing and describing endurants:
 - ⊗ gaseous or
 - ⊗ liquid materials
 - ⊗ being contained in parts,
 - ◇ and perdurants
 - ⊗ actions,
 - ⊗ events and
 - ⊗ behaviours.
- We shall not cover these here, but refer to
 - ◇ [10, Manifest Domains: Analysis & Description] and
 - ◇ [16, From Domains to Requirements].

3.2.8. Domain Science

- There are a number of issues that need be researched.

3.2.8.1 A Prompt Semantics:

- The analysis and description prompts
 - ❖ need be precisely, that is, mathematically defined.
 - ❖ Such a semantics is a first step towards securing a foundation for our approach.
 - ❖ We refer to [8].

3.2.8.2 Laws of Domain Descriptions:

- A semantics of the analysis and description prompts and thus their applications is expected to satisfy the following law:
 - ❖ Analysing (\mathcal{A}) and/or describing (\mathcal{D}) two otherwise unrelated composite parts, p_i and p_j , shall yield the same results whether p_i is treated before p_j or vice-versa:
 $\mathcal{A}(p_i); \mathcal{A}(p_j)$ and $\mathcal{A}(p_j); \mathcal{A}(p_i)$, respectively
 $\mathcal{D}(p_i); \mathcal{D}(p_j)$ and $\mathcal{D}(p_j); \mathcal{D}(p_i)$.
 - ❖ There are many others such laws.

3.2.8.3 Laws of Domains:

- Given an appropriate domain description
 - ❖ it should be possible to prove certain laws
 - ❖ about that domain.

- **An example:**

- ❖ Assume a railway system
 - ❖ with trains operating according to a timetable
 - ❖ that prescribes train departures from and arrivals at any station
 - ❖ according to a 24 hour cycle,
 - ❖ and assume that all trains function precisely.
- Now we would expect the following law to hold over any 24 hour period:
 - ❖ The of trains *arriving* at a station,
 - ❖ minus the number of trains *ending* their journey at that station,
 - ❖ plus number of trains *starting* their journey at that station,
 - ❖ equals the number of trains *leaving* that station ●



- Physics is characterised by its laws.
- So should man-assisted domains.
- A proper theory of domain description
 - ◇ should invite domain laws
 - ◇ to be identified
 - ◇ and proved.
- There is a rich world “*out there*”.

3.2.9. What Can Be Described ?

- Even if we limit ourselves to physically manifest domains [10],
 - ❖ that is, entities that we can observe, i.e., see,
 - ❖ in cases even touch,
 - ❖ there are such which we do not yet know how to describe objectively,
 - ❖ that is, mathematically.
- Moreover, we cannot give a precise delineation of which domains, or aspects of domains, are describable.

- **An example:**

- ❖ We have described aspects of a pipeline system, Sect. 3.2.6.
- ❖ We have even postulated (implementable) functions for observing the flow and leaks of the material (oil, gas, or other) conducted by pipeline units.
- ❖ We also know, but do not show, how to formalise the fluid dynamics of these flows, namely in terms of partial differential equations based on Bernoulli and Navier–Stokes models through individual pipeline units.
- ❖ But we have yet to show how to combine our “discrete mathematics” models with those of fluid dynamics.

- One problem here is
 - ❖ that our discrete mathematics descriptions
 - ⊗ model an infinite variety of pipelines,
 - ⊗ that is, arbitrary compositions of pipeline units,
 - ❖ whereas, conventionally, PDEs,
 - ⊗ model the dynamics only of specific, single units.
 - ❖ It has been suggested that perhaps the Wiener–Feynman–Dirac–Wheeler concept of *Path Integrals*. may be a way to solve the problem ●

- *Our domain models are just abstractions!*
 - ❖ One cannot expect any domain description to “completely” model a domain.
 - ❖ There are simply too many properties to describe.
 - ❖ And there are domain properties that we can informally describe in words, but cannot yet formalise.
 - ❖ Domain description is (therefore) a matter of choice,
 - ⊗ of abstraction level
 - ⊗ and of what to include in the description
 - ⊗ and what to leave out!

3.3. Requirements Engineering

- We would not advocate the *TripTych* to software development
 - ❖ unless we had a method for “deriving” requirements
 - ❖ from domain descriptions.
- And from formal requirements prescriptions
 - ❖ we know how to design software
 - ❖ such that $\mathcal{D}, \mathcal{S} \models \mathcal{R}$,
 - ❖ that is:
 - ⊗ the *Software*
 - ⊗ can be proved correct
 - ⊗ — in the context of the *Domain* —
 - ⊗ with respect to the *Requirements*.

3.3.1. Three Kinds of Requirements

- Our approach to the “derivation” of requirements is based on the following decomposition of requirements into three kinds:
 - ❖ *domain requirements*,
 - ❖ *interface requirements* and
 - ❖ *machine requirements*
- where the *machine* is the
 - ❖ hardware and
 - ❖ softwareto be developed

3.3.2. Domain Requirements

- By *domain requirements*
 - ❖ we shall understand such requirements
 - ❖ that can be expressed
 - ❖ solely using terms of the domain
 - ⊗ that is, terms defined in
 - ⊗ the domain description.

- The “derivation”
 - ❖ of domain requirements prescriptions
 - ❖ from domain descriptions
 - ❖ is governed by a set of “derivation” operations.
- Examples of these ‘derivation’ operations are:
 - ❖ *projection*,
 - ❖ *instantiation*,
 - ❖ *determination*,
 - ❖ *extension* and
 - ❖ *fitting*.

- *Projection* means that we remove
 - ❖ from the evolving requirements prescription
 - ❖ those entity descriptions of the domain
 - ❖ which are not to be considered
 - ❖ when (further) prescribing the requirements.
- **An example:**
 - ❖ From the example of the road net and traffic system we remove the vehicles and the monitor ●

- *Instantiation* means that we concretise, i.e., prescribe “less-abstract”,
 - ❖ those retained domain phenomena
 - ❖ whose concretisation
 - ❖ it is suitable to prescribe.
- **An example:** The general road net
 - ❖ is instantiated to a “linear” toll-road system
 - ❖ of a sequence of toll-road hubs
 - ❖ connected, “up” and “down” the toll-road to neighbouring toll-road hubs,
 - ❖ and, by means of toll-road plazas, to a remaining road net ●

- What do we mean by: “it is suitable to prescribe” ?
 - ⋄ Well, first of all, we have to realize the following:
 - ⊗ requirements must only prescribe what can be computed.
 - ⊗ That means that entities whose realisability
 - ⊗ in terms of computable data structure or functions
 - ⊗ must eventually be so prescribed.

- ❖ Secondly, as requirements prescription may, and normally will
 - ⊗ proceed in stages,
 - ⊗ one (i.e., the requirements engineer) may decide
 - ⊗ to instantiate some entities
 - ⊗ while leaving other entities “untouched”,
 - ⊗ only to return to the concretisation of these in a later stage.
- ❖ And so forth.
- ❖ It is all a matter of style and taste!

- *Determination* means that there may be
 - ◇ entities, i.e., endurants or perdurants, that are described to be non-deterministic in the domain
 - ◇ but which, after projection and instantiation
 - ◇ need be prescribed to be “less non-deterministic”.
 - ◇ **An example:**
 - ⊗ Whereas hubs
 - * in general allow traffic
 - * from any link incident upon that hub
 - * to any links emanating from that hub
 - * but so that signaling, as expressed in the hub states,
 - * may, at times, prevent some emanating links to be accessible from some incident links;
 - ⊗ a toll-road hub,
 - * in order to be an appropriate toll-road,
 - * must allow for free flow
 - * from any incident link
 - * to any emanating link ●

- *Extension* typically means
 - ⊠ that there may be entities
 - ⊠ that were “hitherto” not computationally feasible,
 - ⊠ but where
 - * new technologies
 - * or higher labour costs
 - ⊠ mandate their feasibility —
 - ⊠ thus making way for introducing these new technologies
 - ⊠ into a this ‘extended’ domain.
 - ⊠ **An example** is that of
 - ⊠ the electronic sensing of vehicles
 - ⊠ entering or leaving a toll-road —
 - ⊠ thus enabling “*road pricing*” ●

- *Fitting* is necessitated
 - ⊠ when two or more requirements projects
 - ⊠ based on “the same” domain,
 - ⊠ and with “overlapping domain coverage”
 - ⊠ need be “harmonised”.
 - ⊠ **An example:**
 - ⊠ One set of requirements are being prescribed for a *road state-of-repair and maintenance facility*,
 - ⊠ another set of requirements are being prescribed for a *road pricing system*.
 - ⊠ Now they must both rely on some sort of representation of the same road net ●

3.3.3. Interface Requirements

- By *interface requirements*
 - ❖ we shall understand such requirements
 - ❖ that can be expressed
 - ❖ only using terms both
 - ⊗ of the domain and
 - ⊗ of the machine.

- In order to structure the interface requirements
 - ❖ we introduce a notion of *shared phenomena*
 - ❖ whether endurants or perdurants.
 - ❖ If a phenomenon is present in the domain
 - ❖ and if it is also to be present in the machine to be designed
 - ❖ then that phenomenon is said to be shared.
- As a result we structure interface requirements prescriptions around
 - ❖ *shared endurants*,
 - ❖ *shared actions*,
 - ❖ *shared events* and
 - ❖ *shared behaviours*.

- *Shared endurants*

- ❖ pose two “problems”
 - ⊗ the initialisation of endurant data structures and their values,
 - ⊗ and the regular access to and update of endurant data.
- ❖ Both must be prescribed.
- ❖ Usually both require
 - ⊗ the interaction between
 - ⊗ the domain and the machine.
- ❖ **An example:**
 - ⊗ Road nets are shared between the domain and the machine.
 - ⊗ Initially all hubs and all links need be structured in some data structure, say a database.
 - ⊗ The shared endurant requirements must now specify which, usually composite database operations
 - * are to be used in establishing the database, and
 - * which are to be used in accessing and updating the endurants.

- *Shared actions*

- ❖ imply an interaction between
 - ⊗ between the domain
 - ⊗ and the machine.
- ❖ That interaction is typically manifested
 - ⊗ by interaction between
 - * either humans of the domain
 - * or physical domain entities
 - ⊗ and the machine

❖ Example: Human/Machine Interaction:

- ⊗ The payment of a road price fee today involves a human (say, with a credit card)
- ⊗ and the machine, checking and accepting or rejecting the credit card, etcetera ●

❖ Example: Machine/Machine Interaction:

- ⊗ The electronic recording (within the machine)
- ⊗ of a vehicle passing a toll-gate barrier (another part of the machine)
- ⊗ and the vehicle itself (another machine, external to required machine) ●

- And so on, for
 - ❖ *shared events* and
 - ❖ *shared behaviours*.

3.3.4. Machine Requirements

- By *machine requirements*
 - ❖ we shall understand such requirements
 - ❖ that can be expressed
 - ❖ solely using terms of the machine.
- Since that is the case:
 - ❖ no “mention” of the domain
 - ❖ in the machine requirements
 - ❖ we shall omit covering this field.

3.4. Discussion

- We have suggested that
 - ❖ there are a set of principles and techniques
 - ❖ for “deriving” a major set of requirements
 - ❖ from domain descriptions.
- This, then, is an argument for
 - ❖ taking domain modelling serious:
 - ❖ there are principles and techniques
 - ❖ for bringing you
 - ⊗ from domain descriptions
 - ⊗ to requirements prescriptions
 - ⊗ and from there on to software design.
 - ⊗ We refer to [4, 16, 2008–2015] for details.

Concluding Discussion

- We claim to have justified our claim
 - ❖ that *Software* must be *designed* on the basis
 - ❖ of *Requirements prescriptions* that have been “derived”
 - ❖ from *Domain descriptions*,
 - ❖ all of them formally.
- In this way we can secure that software
 - ❖ fulfill users’/customers’ expectations
 - ⊗ since the requirements
 - ⊗ are strongly related to the domain
 - ❖ and is correct: $\mathcal{D}, \mathcal{S} \models \mathcal{R}$.
- It is the only way in which we can see these two, *expectations* and *correctness*, fulfilled.

4.1. Papers on Domain Science & Engineering

- I mention but a few of my earlier papers related to domain science & engineering.
- [10, *Manifest Domains: Analysis & Description, 2014*] is the definitive paper on domain analysis and description.
- [16, *From Domains to Requirements — A Different View of Requirements Engineering, 2015*] is the definitive paper on “derivation” of requirements prescriptions from domain descriptions. It is based, in part, on [4, *From Domains to Requirements*].
- [5, *Domain Engineering, 2008*] treats an aspect of domain modelling referred to as *domain facets*. We expect to revise [5].
- [6, *Domains: Their Simulation, Monitoring and Control, 2011*]. The concepts of simulation, monitoring and simulation are analysed in the light of the domain–requirements–design *TripTych*.

- [7, *A Rôle for Mereology in Domain Science and Engineering, 2009*]. Stanisław Leśniewski's replacement of Bertrand Russell's set theory axiomatisation is reviewed and it is shown how part/sub-part relations can be interpreted as a relation between (Hoare) CSP-processes.
- [9, *Domain Engineering – A Basis for Safety Critical Software. 2014*]. Issues of *system safety criticality* that can be considered already before requirements engineering are here seen in the light of domain engineering.

4.2. A Research and Experimental Engineering Programme

- In the papers on which the current paper is based a number of open problems have been identified.

4.2.1. The Mathematics of Analysis & Description Prompts

- In [8, *Domain Analysis: Endurants – An Analysis & Description Process Model*] we present a formal semantics of the analysis and description process.
- And in [11, *Domain Analysis: Endurants – a Consolidated Model of Prompts*] we are extending this model to also cover the meaning of the prompts themselves.
- These are models of the informal “worlds” of domains.
- The study of this area is elusive.

4.2.2. Analysis & Description Calculi for Other Domains

- The analysis and description calculus of this paper appears suitable for manifest domains.
- For other domains other calculi appears necessary.
 - ⋄ There is the introvert, composite domain of systems software:
 - ⊗ operating systems, compilers, database management systems, Internet-related software, etcetera.
 - ⊗ The classical computer science and software engineering disciplines related to these components of systems software appears to have provided the necessary analysis and description “calculi.”

- ❖ There is the domain of financial systems software
 - ⊗ accounting & bookkeeping,
 - ⊗ banking systems,
 - ⊗ insurance,
 - ⊗ financial instruments handling (stocks, etc.),
 - ⊗ etcetera.
- Etcetera.
- For each domain characterisable by a distinct set of analysis & description calculus prompts such calculi must be identified.
- It seems straightforward:
 - ❖ to base a method for analysing & describing a category of domains
 - ❖ on the idea of prompts like those developed in this lecture.

4.2.3. On Domain Description Languages

- We have in this seminar expressed the domain descriptions in the **RAISE** [25] specification language **RSL** [24].
- With what is thought of as basically inessential, editorial changes, one can reformulate these domain description texts in either of
 - ❖ **Alloy** [28] or
 - ❖ **The B-Method** [1] or
 - ❖ **VDM** [19, 20, 23] or
 - ❖ **Z** [34].
- One could also express domain descriptions algebraically, for example in **CafeOBJ**.
 - ❖ The analysis and the description prompts remain the same.
 - ❖ The description prompts now lead to **CafeOBJ** texts.

- We did not go into much detail with respect to perdurants, let alone behaviours.
 - ❖ For all the very many domain descriptions, covered elsewhere, **RSL** (with its **CSP** sub-language) suffices.
 - ❖ But there are cases where we have conjoined our **RSL** domain descriptions with descriptions in
 - ⊗ **Petri Nets** [32] or
 - ⊗ **MSC** [27] or
 - ⊗ **StateCharts** [26].
- Since this seminar only focused on endurants there was no need, it appears, to get involved in temporal issues.
- When that becomes necessary, in a study or description of perdurants, then we either deploy
 - ❖ **DC: The Duration Calculus** [35] or
 - ❖ **TLA+: Temporal Logic of Actions** [30].

4.2.4. Commensurate Discrete and Continuous Models

- The pipeline example hinted at
 - ⋄ co-extensive descriptions of discrete and continuous behaviours,
 - ⋄ the former in, for example, **RSL**,
 - ⋄ the latter in, typically, the calculus mathematics of partial differential equations (**PDEs**).
 - ⋄ The problem that arises in this situation is the following:
 - ⊗ there will be, say variable identifiers, e.g., x, y, \dots, z
 - ⊗ which in the **RSL** formalisation has one set of meanings, but
 - ⊗ which in the **PDE** “formalisation” has another set of meanings.

- ❖ Current formal specification languages⁴ do not cope with continuity.
- Some research is going on.
- But to substantially cover, for example, the proper description of laminar and turbulent flows in networks (e.g., pipelines) requires more substantial results.

⁴Alloy [28],
Event B [1],
RSL [24],
VDM-SL [19, 20, 23],
Z, etc.

4.2.5. Interplay between Parts and Materials

- The pipeline example revealed but a small fraction of the problems that may arise in connection with modeling the interplay between parts and materials.
- Subject to proper formal specification language and, for example PDE specification we may expect more interesting
 - ❖ laws, as for example those of pipeline flows
 - ❖ and even proof of these as if they were theorems.
- Formal specifications have focused on verifying properties of requirements and software designs.
- With co-extensive (i.e., commensurate) formal specifications of both discrete and continuous behaviours we may expect formal specifications to also serve as bases for predictions.

4.2.6. The Mathematics of Domain-to-Requirements Operators

- In [16, *From Domains to Requirements – A Different View of Requirements Engineering*]⁵
 - ⊠ we postulate that certain properties hold
 - ⊠ between domain requirements prescriptions
 - ⊠ “before” and “after”
 - ⊠ the application of the domain-to-requirements operations:
 - ⊠ *projection*,
 - ⊠ *instantiation*,
 - ⊠ *determination*,
 - ⊠ *extension* and
 - ⊠ *fitting*.
- These postulated properties need be studied further.

⁵[16] is a rather extensive revision of [4].

4.2.7. Further Work on Domain-to-Requirements and Interface Techniques

- In [16, *From Domains to Requirements – A Different View of Requirements Engineering*]
 - ❖ we have shown a number of techniques
 - ❖ for domain-to-requirements operations,
 - ❖ in particular those that yield domain requirements.
- In [16] we also show some techniques
 - ❖ that pertain to interface requirements,
 - ❖ but it seems more study is required.

4.3. Tony Hoare's Summary on 'Domain Modeling'

- In a 2006 e-mail, in response, undoubtedly to my steadfast, perhaps conceived as stubborn insistence, on domain engineering,
- Tony Hoare summed up his reaction to domain engineering as follows, and I quote⁶:

“There are many unique contributions that can be made by domain modeling.

- 1 The models describe all aspects of the real world that are relevant for any good software design in the area. They describe possible places to define the system boundary for any particular project.
- 2 They make explicit the preconditions about the real world that have to be made in any embedded software design, especially one that is going to be formally proved.

⁶E-Mail to Dines Bjørner, July 19, 2006

- 3 They describe the whole range of possible designs for the software, and the whole range of technologies available for its realisation.
- 4 They provide a framework for a full analysis of requirements, which is wholly independent of the technology of implementation.
- 5 They enumerate and analyse the decisions that must be taken earlier or later in any design project, and identify those that are independent and those that conflict. Late discovery of feature interactions can be avoided.”

4.4. A Re-evaluation of Computer and Computing Science

- By **computer science** we understand
 - ❖ the study and knowledge about
 - ❖ the phenomena that can “exist inside” computers.
- By **computing science** we understand
 - ❖ the study and knowledge about
 - ❖ how to construct those phenomena.

- If we accept the *TripTych* dogma
 - ❖ of basing software design
 - ❖ on precise requirements prescriptions
 - ❖ which are based on precise domain descriptions,
- then training, teaching and research in
 - ❖ computer and
 - ❖ computing science
- must be revised.

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